EXERCISE – IV ADVANCED SUBJECTIVE QUESTIONS

If C_0, C_1, C_2, ... , C_n are the combinatorial coefficients in the expansion of $(1 + x)^n$, $n \in N$, then prove the following :

1. $C_0^2 + C_1^2 + C_2^2 + \ldots\ldots + C_n^2 = \dfrac{(2n)!}{n!\,n!}$

(This result is to be remembered)

Sol.

2. $C_0 C_1 + C_1 C_2 + C_2 C_3 + \ldots. + C_{n-1} C_n = \dfrac{(2n)!}{(n+1)!\,(n-1)!}$

Sol.

3. $C_1 + 2C_2 + 3C_3 + \ldots\ldots\ldots + n \,.\, C_n = n \,.\, 2^{n-1}$

Sol.

4. $C_0 + 2C_1 + 3C_2 + \ldots\ldots\ldots + (n + 1)C_n = (n + 2)2^{n-1}$

Sol.

5. $C_0 + 3C_1 + 5C_2 + \ldots\ldots\ldots + (2n + 1)C_n = (n + 1)\, 2^n$

Sol.

6. $(C_0 + C_1)(C_1 + C_2)(C_2 + C_3) \ldots (C_{n-1} + C_n) = \dfrac{C_0.C_1.C_2.\ldots\ldots.C_{n-1}(n+1)^n}{n!}$.

Sol.

7. If P_n denotes the product of all the coefficients in the expansion of $(1 + x)^n$, $n \in N$, show that, $\dfrac{P_{n+1}}{P_n} = \dfrac{(n+1)^n}{n!}$.

Sol.

8. $\frac{C_1}{C_0}+\frac{2C_2}{C_1}+\frac{3C_3}{C_2}+.....+\frac{n.C_n}{C_{n-1}}=\frac{n(n+1)}{2}$

Sol.

9. $C_0 + \frac{C_1}{2}+\frac{C_2}{3}+.....+\frac{C_n}{n+1}=\frac{2^{n+1}-1}{n+1}$

Sol.

10. $2.C_0 + \frac{2^2.C_1}{2}+\frac{2^3.C_2}{3}+\frac{2^4.C_3}{4}+......+\frac{2^{n+1}.C_n}{n+1}=\frac{3^{n+1}-1}{n+1}$

Sol.

11. $C_0C_r + C_1C_{r+1} + C_2C_{r+2} + ... + C_{n-r}C_n = \frac{2n!}{(n-r)!(n+r)!}$

Sol.

12. $C_0 - \frac{C_1}{2} + \frac{C_2}{3} - .... + (-1)^n \frac{C_n}{n+1} = \frac{1}{n+1}$

Sol.

13. $C_0 - 2C_1 + 3C_2 - 4C_3 + ... + (-1)^n (n + 1) C_n = 0$

Sol.

14. $(n - 1)^2 . C_1 + (n - 3)^2 . C_3 + (n - 5)^2 . C_5 + ...$

$........ = n (n + 1) 2^{n - 3}$

Sol.

15. $1 . C_0^2 + 3 . C_1^2 + 5 . C_2^2 + ... + (2n + 1) C_n^2 = \frac{(n+1)(2n)!}{n!\,n!}$

Sol.

16. If a_0 , a_1 , a_2 , ... be the coefficients in the expansion of $(1 + x + x^2)^n$ in ascending powers of x, then prove that

Sol.

(i) $a_0 a_1 - a_1 a_2 + a_2 a_3 - .... = 0$

Sol.

(ii) $a_0a_2 - a_1 a_3 + a_2 a_4 - ..... + a_{2n-2} a_{2n} = a_{n+1}$ or a_{n-1}.

Sol.

(iii) $E_1 = E_2 = E_3 = 3^{n-1}$ where
$E_1 = a_0 + a_3 + a_6 + ...$; $E_2 = a_1 + a_4 + a_7 + ...$
& $E_3 = a_2 + a_5 + a_8 + ...$

Sol.

17. Prove that : $\sum_{r=0}^{n-2}({}^nC_r . {}^nC_{r+2}) = \frac{(2n)!}{(n-2)!\,(n+2)!}$

Sol.

18. If $(1 + x)^n = C_0 + C_1 x + C_2x^2 + \ldots\ldots\ldots + C_nx^n$, then show that the sum of the products of the C_i' s taken two at a time, represented by $\underset{0\ \le i<j\le n}{\Sigma\ \Sigma}\, C_i\, C_j$ is equal to $2^{2n-1} - \frac{2n!}{2(n!)^2}$.

Sol.

19. $\sqrt{C_1} + \sqrt{C_2} + \sqrt{C_3} + \ldots\ldots + \sqrt{C_n} \le 2^{n-1} + \frac{n-1}{2}$.

Sol.

20. $\sqrt{C_1} + \sqrt{C_2} + \sqrt{C_3} + \ldots\ldots + \sqrt{C_n} \le \left[n(2^n - 1)\right]^{1/2}$ for $n \ge 2$.

Sol.